

# Randomness Evaluation and Hardware Implementation of Nonadditive CA-Based Stream Cipher

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## Abstract

We shall review the cellular automaton (CA)-based pseudorandom-number generators (PRNGs), and show that one of these PRNGs can generate high-quality random numbers which can pass all of the statistical tests provided by the National Institute of Standards and Technology (NIST). A CA is suitable for hardware implementation. We demonstrate that the CA-based stream cipher, which is implemented in the field-programmable gate arrays (FPGA), has a high encryption speed in a real-time video encryption and decryption system.

KEYWORDS: Pseudorandom-Number Generator, Cellular Automata, Statistical Test, FPGA Implementation, Real-time Encryption

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## 1 Introduction

Many secret key cryptosystems have been proposed thus far [1]. One of the advantages of a stream cipher over a block cipher is its high encryption speed and small gate size when it is implemented in hardware. Therefore, a stream cipher is suitable for developing high-speed and low-power encryption systems. There will be an increase in demand for the development of faster encryption associated with high-resolution video, high-volume data retrieval, and other high-speed data communication systems such as 10 Gbit networks. Thus, in various fields, a hardware-like stream cipher is now requested for real-time encryption and decryption.

In this paper, we propose a CA-based PRNG used for a stream cipher which is suitable for hardware implementation because of its simple construction (i.e., locality of interaction and homogeneous units) [2]. A one-dimensional elementary cellular automaton (ECA) consists of a line of cells with  $S_i = 0$  or 1 for  $i = 0, 1, 2, \dots, N$ . These cell values are updated in parallel in discrete time steps according to a fixed rule of the form,

$$S_i^{t+1} = F(S_{i-1}^t, S_i^t, S_{i+1}^t) \quad (1)$$

where  $S_i^t$  denotes the  $i$  cell value at time  $t$ . Wolfram firstly used the ECA as a PRNG, and investigated its randomness [3]. He concluded that the following ‘rule 30’ is the best PRNG among the ECA rules,

$$S_i^{t+1} = S_{i-1}^t \oplus S_i^t \oplus S_{i+1}^t \oplus S_i^t \cdot S_{i+1}^t \quad (2)$$

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or

$$S_i^{t+1} = S_{i-1}^t \text{ XOR } (S_i^t \text{ OR } S_{i+1}^t) \quad (3)$$

where  $\oplus$  denotes plus modulo 2. He also proposed the CA-based stream cipher using this ECA30 [4]. It is known that ECA30 has large periodic cycles. The maximum period is  $2^{0.6N}$  with systems size  $N$ . Most configurations fall into the cycle if we set the system size  $N$  sufficiently large. Now, Wolfram also emphasizes that ECA30 is the ‘origin of randomness’ in his new book [5].

On the other hand, additive CA-based PRNGs, such as rule 90, rule 150, rule 105, and rule 165,<sup>1</sup> have been proposed by Hortensius et al. [6, 7], Nandi and Chaudhuri [8, 9], and Tomassini et al. [10, 11, 12]. Tomassini has proposed that rule 165 is the best PRNG among the ECA rules. Moreover, CA with rule 90, rule 105, rule 150, and rule 165 is the best PRNG among the inhomogeneous CA rules. This is because Tomassini et al evaluated the randomness using the results of the Diehard test suite [13] which does not have a linear complexity test. We originally found that these CAs do not pass the linear complexity test which is one of the NIST statistical tests [14]. The linear complexity test is crucial for the application of a PRNG to a cryptosystem because this test detects whether the prediction is possible. It is also known that linear CA is equivalent to the linear feedback shift register of the same size even if we use these rules inhomogeneously [15, 16]. In fact, Nandi and Chaudhuri proposed an additive CA-based block cipher with nonlinear transformations [17] after realizing this point [18, 19]. Mihaljevic and Cattell also independently proposed additive CA-based cryptosystems [20, 21, 22, 23].

Guan et al. proposed a new class of CA (controllable CA and two-dimensional CA with an asymmetric neighborhood), and investigated their randomness using the Diehard test suite [24, 25]. In this paper, we investigate the randomness of sequences generated by nonadditive CAs, that are ECA30 and its 5-neighbor extension (rule 535945230 in 5-neighbor CA framework), using the statistical test suite provided by NIST, and compared them with some good PRNGs (AES, SHA1, and MUGI). After we show the hardware implementation of these CAs in FPGA, we demonstrate that these CAs have a high encryption speed in experiments of real-time video encryption and decryption systems.

## 2 Randomness Evaluation

Randomness is one of the crucial points for a keystream of secure stream ciphers. Although various types of statistical test for randomness have been proposed thus far [13, 26, 27], we will focus on the NIST statistical test suite [14], and will show the results of this test suite.

### 2.1 On NIST statistical test suite

The NIST statistical test suite is a statistical package consisting of 16 tests that were developed to test the randomness of arbitrary long binary sequences produced by either hardware or software-based cryptographic random- or pseudorandom-number generators. These tests focus on different types of non-randomness that could exist in a sequence. The 16 tests are listed in Table 1. Note that the test settings of discrete fourier transform test and Lempel Ziv compression test are wrong [28]. So, in what follows, we use the corrected version of the test suite [29].

For each statistical test, a set of P-values, which corresponds to the set of sequences, is produced. Each sequence is called *success* if the corresponding P-value satisfies the condition  $\text{P-value} \geq \alpha$ , and is otherwise called *failure*. For a fixed significance level  $\alpha$ ,  $100\alpha$  % of P-values are expected to indicate failure<sup>2</sup>. For the interpretation of test results, NIST adopts the following two approaches,

- (1) the examination of the proportion of success sequences (success rate)

If the proportion of success sequences falls outside of the following acceptable interval, there is evidence that the data is nonrandom.

$$R \pm 3\sqrt{\frac{R(1-R)}{m}} \quad (4)$$

Here,  $R = 1 - \alpha$  and  $m$  is the number of sequences. This interval is determined to be in the 99.73% range of the normal distribution which is an approximation of the binomial distribution under the assumption that each sequence is an independent sample.

<sup>1</sup>Rules of linear CA have only XORs, and rules of additive CA have only XOR and XNOR.

<sup>2</sup>All the statistical tests of the NIST statistical test suite have the unique significance level  $\alpha = 0.01$ .

Table 1: List of the NIST Statistical Tests

Number	Test Name
1	Frequency
2	Block Frequency
3	Runs
4	Longest Run
5	Binary Matrix Rank
6	Discrete Fourier Transform
7	Non-overlapping Template Matching
8	Overlapping Template Matching
9	Universal
10	Lempel Ziv Compression
11	Linear Complexity
12	Serial
13	Approximate Entropy
14	Cumulative Sums
15	Random Excursions
16	Random Excursions Variant

(2) the examination of the uniformity of the distribution of P-values

This examination is accomplished by computing the following  $\chi^2$  value.

$$\chi^2 = \sum_{i=1}^{10} \frac{(F_i - m/10)^2}{m/10} \quad (5)$$

Here,  $F_i$  is the number of P-values in subinterval  $[(i-1)*0.1, i*0.1)$ , and  $m$  is the number of sequences (sample size). The P-value of P-values is calculated such that  $P'$ -value = **igamc** ( $9/2, \chi^2/2$ ), where **igamc**(n,x) is the incomplete gamma function. If  $P'$ -value  $\geq 0.0001$ , then the set of P-values can be considered to be uniformly distributed.

## 2.2 Test results

In this subsection, we show the results of the NIST statistical test suite for several PRNGs. For each statistical test, the two analyses described above are executed, and evaluated whether the set of sequences *passes* the test. We used 1000 samples of  $10^6$  bit sequences for each test. Consequently,  $10$  (keys)  $\times$   $1000$  (sample)  $\times$   $10^6$  (sequence) bits are used for each test in order to investigate the difference in results between different keys<sup>3</sup>. The input parameters that we used are listed in Table 2. In the CA case, we used the cell values  $\{S_i^t\}$  with a fixed cell number  $i$  as a keystream, and also used the system size  $N = 1000$  and periodic boundary condition.

### Results of ECA30

Table 3 shows the results of ECA 30. While all tests are passed in the best cases (key 4, key 5, key 7 and key 10), the runs test (number 3), the non-overlapping template matching test (number 7), the random excursions test (number 15), and the random excursion variance test (number 16) fail in the worst case (key 1). The success rates of the worst case (key 1) and of the best case (key 4) are shown in Figure 1. Solid lines denote the acceptable interval specified by eq.(4). As we can see, some tests have many success rates. For example, the non-overlapping template matching test (number 7) has 148 success rates because one success rate corresponds to one-template (nonperiodic pattern consisting of 9 bits) matching. If at least one success rates is out of the acceptable interval, then the test fails (see key 1 case).

<sup>3</sup>The key is the initial configuration  $\{S_i^{t=0}\}$  in the CA case.

Table 2: Parameters used for the NIST Test Suite

Test Name	Block Length
Block Frequency	20,000
Non-overlapping Template Matching	9
Overlapping Template Matching	9
Universal (Initialization Steps)	7 (1280)
Linear Complexity	500
Serial	10
Approximate Entropy	10

Table 3: Results of ECA30. *Pass* denotes a set of sequences that passed all 16 tests. The other numbers denote the failed test number listed in Table 1.

Key	Success Rate	Uniformity
1	3, 7, 15, 16	pass
2	15, 16	pass
3	7	pass
4	pass	pass
5	pass	pass
6	7	pass
7	pass	pass
8	7	pass
9	8	pass
10	pass	pass

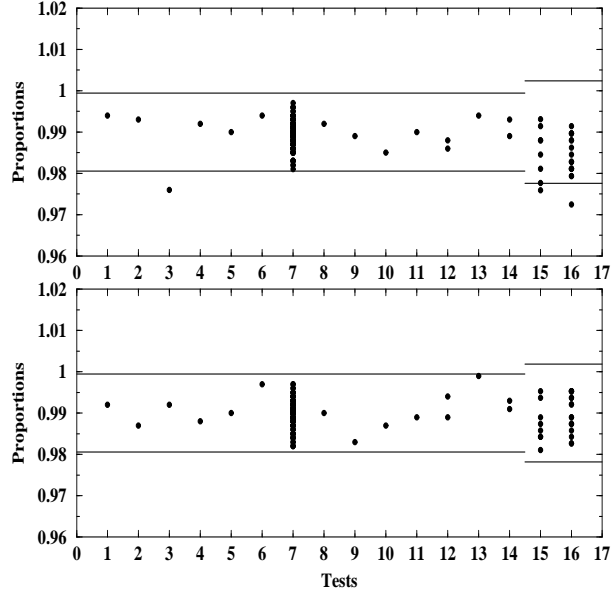
Figure 1: Success rates of ECA30 for 16 tests. Key 1 and key 4 cases are shown in up and down figures, respectively. Solid lines denote the acceptable interval (eq.(4) with  $\alpha = 0.01$ ).

Table 4: Results of ECA30 with rotation shift (11 cells)

Key	Success Rate	Uniformity
1	pass	pass
2	pass	pass
3	7	pass
4	7	pass
5	pass	pass
6	7	pass
7	pass	pass
8	7	pass
9	7	pass
10	pass	pass

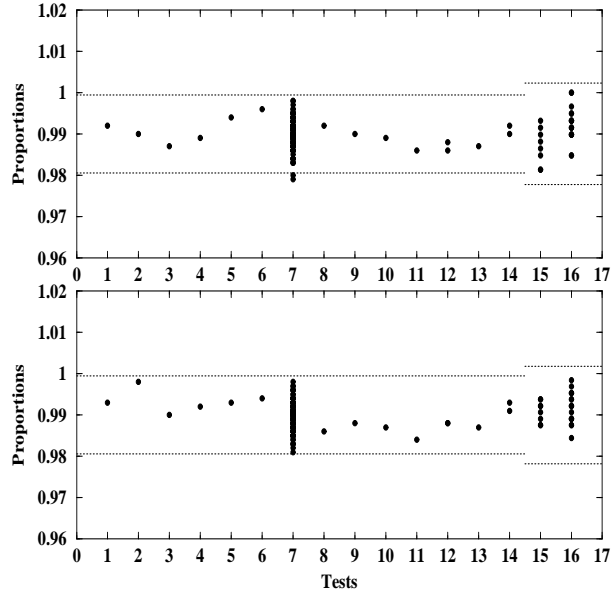


Figure 2: Success rates of ECA30 with rotation shift (11 cells). Key 9 and key 10 cases are shown in up and down figures, respectively. Solid lines denote the acceptable interval (eq.(4) with  $\alpha = 0.01$ ).

Table 5: Results of AES

Key	Success Rate	Uniformity
1	pass	pass
2	pass	pass
3	15	pass
4	pass	pass
5	7	pass
6	14	pass
7	7, 8	pass
8	pass	pass
9	pass	pass
10	pass	pass

Table 6: Results of SHA1

Key	Success Rate	Uniformity
1	pass	pass
2	pass	pass
3	7	pass
4	7	pass
5	pass	pass
6	7, 15, 16	pass
7	7	pass
8	7	pass
9	pass	pass
10	pass	pass

We investigated the test results in the cases that we added rotation shift to ECA30 in each time step. Table 4 shows the results of ECA30 with rotation shift (11 cells). The success rates of the worst case (key 9) and of the best case (key 10) are shown in Figure 2. This time, all tests pass in the five cases (key 1, key 2, key 5, key 7 and key 10). It seems that the randomness of sequences is slightly improved. Although the non-overlapping template matching test fails in five cases, the number of templates whose success rate is out of the acceptable interval (but very close to the boundary) is only one or two. However, we found that rotation shift does not always improve the randomness of sequences effectively.

### Results of good PRNGs

Tables 5, 6, and 7 show the results of AES (128 bit key, OFB mode), SHA1, and MUGI, respectively, in order to compare the results between ECA30 and them. As we can see, all tests are passed in six cases (AES), in five cases (SHA1), and in seven cases (MUGI), respectively. Note that the SHA1 case is the same frequency as the ECA30 with rotation shift (11 cell).

### Results of 5-neighbor CA

We can obtain the following equation if we consider two iterations of eq.(2),

$$\begin{aligned}
S_i^{t+1} = & S_{i-2}^t \oplus S_{i+1}^t \oplus S_{i+2}^t \oplus \\
& S_{i-1}^t \cdot S_{i+1}^t \oplus S_{i-1}^t \cdot S_{i+2}^t \oplus S_i^t \cdot S_{i+1}^t \oplus \\
& S_i^t \cdot S_{i+2}^t \oplus S_{i+1}^t \cdot S_{i+2}^t \oplus \\
& S_{i-1}^t \cdot S_{i+1}^t \cdot S_{i+2}^t \oplus S_i^t \cdot S_{i+1}^t \cdot S_{i+2}^t
\end{aligned} \tag{6}$$

This is equivalent to rule 535945230 in the 5-neighbor CA framework [5]. We have investigated the

Table 7: Results of MUGI

Key	Success Rate	Uniformity
1	7	pass
2	pass	pass
3	pass	pass
4	pass	pass
5	7	pass
6	pass	pass
7	pass	pass
8	pass	pass
9	7	pass
10	pass	pass

Table 8: Results of CA5-535945230 with rotation shift (11 cells)

Key	Success Rate	Uniformity
1	pass	pass
2	7, 12	pass
3	pass	pass
4	pass	pass
5	7	pass
6	pass	pass
7	pass	pass
8	7	pass
9	pass	pass
10	7	pass

randomness of sequences generated by some class of 5-neighbor CA rules. We found that rule 535945230 is the best.

Table 8 show the results of CA5-535945230 with rotation shift (11 cells). We use one cell  $S_i^t$  (fixed  $i$ ) as a keystream at each time step as well as ECA30 cases. As we can see, all tests are passed in six cases. This is the same frequency as AES. We can conclude that the CA5-535945230 with rotation shift (11 cells) has good randomness, which is comparable to well-known good PRNGs such as AES, SHA1, and MUGI.

### 2.3 Security discussion

It is known that ECA30-based stream cipher which was proposed by Wolfram has a security problem. If we use two consecutive cell values  $(S_i, S_{i+1})$  as a keystream at each time step, an attacker can easily calculate the secret key (initial configuration) from the keystream using the following equation which is the same equation as eq.(2).

$$S_{i-1}^t = S_i^{t+1} \oplus S_i^t \oplus S_{i+1}^t \oplus S_i^t \cdot S_{i+1}^t \quad (7)$$

In order to avoid this, Wolfram proposed that we should use only one cell value  $(S_i)$  as a keystream at each time step. He suggested that an attacker cannot easily calculate the secret key from the keystream in this case (exponential time is required). However, the effective key size is much less than  $N$  even in this case [30]. We should set system size  $N = 2000$  in order to set the effective key size to more than 80 in this Wolfram case.

In order to avoid this attack, we sample cell values such that the distance between consecutively sampled cells becomes larger (e.g., cell numbers 1, 7, 14, 22, 31, 41,  $\dots$ , 932, 976 are sampled for 40 bit

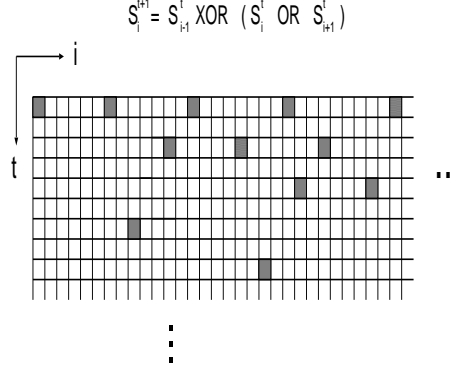


Figure 3: Sampled cells in CA5-535945230 case

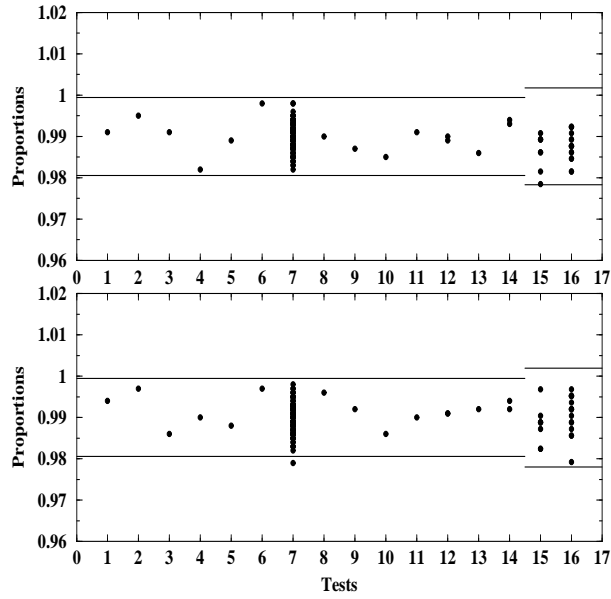


Figure 4: Success rates of ECA30 with sampling method for two different keys

per clock), and rotation shift (11 cells) is added at each time step. As a result, we sample cell values which are denoted as shaded cells in Figure 3 in the CA5-535945230 case if we consider the 3-neighbor CA framework. In this case, the attack mentioned above no longer applies directly. It is difficult to calculate the secret key from the keystream (shaded cells) using eq.(7). If someone could find another attack, the effective key size would be improved as compared with the Wolfram type.

The keystream using this sampling method also has high-quality randomness. The success rates of the ECA30 case and CA5-535945230 case are shown in Figure 4 and Figure 5, respectively. As we can see, all tests are passed except the non-overlapping template matching test (for one template). Note that the linear complexity test is also passed even if we choose the maximum parameter  $M = 5000$  in both cases.

It is well known that statistical characteristics of the keystream are just a component of the security evaluation of a stream cipher. In this paper, we propose an encryption approach based on CA with desirable statistical characteristics and implementation suitability, but that its detailed security evaluation is out of our scope (and that this issue is an open one).



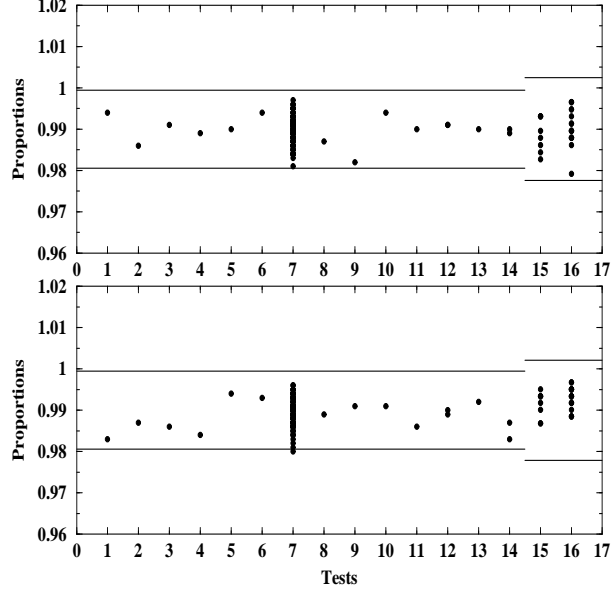


Figure 5: Success rates of CA5-535945230 with sampling method for two different keys

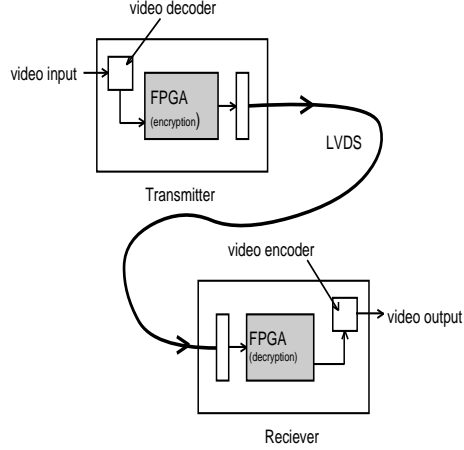


Figure 6: DDR-SDRAM evaluation boards

### 3 Hardware Implementation

Figure 6 shows a schematic of DDR-SDRAM evaluation boards produced by Tokyo Electron Device Ltd. There are video input and output (40 bit per clock), FPGA (VirtexII), and low voltage differential signaling (LVDS) on this board for the purpose of real-time video encryption and decryption. We implemented the CA-based stream cipher on two FPGAs (see Fig. 7), and executed the experiment of real-time video encryption and decryption (see Fig. 8).

#### 3.1 Implementation results

Table 9 shows the implementation results for the system size  $N = 1000$  case. As we can see, both algorithms work up to a high clock frequency because of their simple construction. In the CA5 case, randomness and security level are higher than those in the ECA30 case although encryption speed and gate size are lower. If we set the system size  $N$  larger, the encryption speed becomes higher. On the other hand, if we set the system size  $N$  smaller, the gate size becomes smaller although we have shown only the system size  $N = 1000$  case. Actually, we realized a 1 Gbps encryption speed because the board

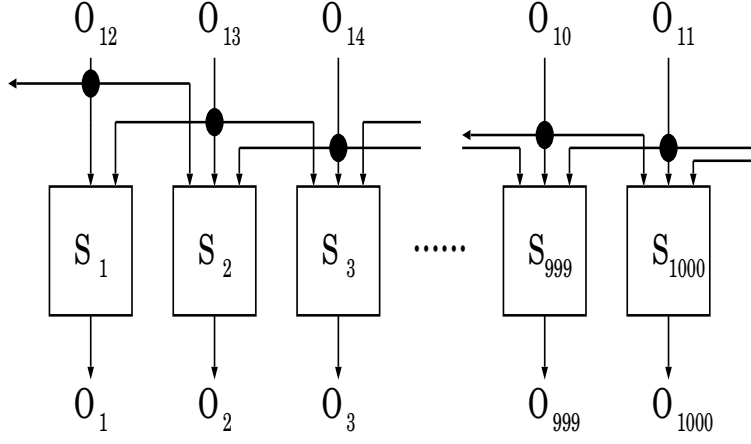


Figure 7: CA-based PRNG core



Figure 8: Real-time video encryption and decryption system

Table 9: Implementation results

	ECA30	CA5
gate size (gate)	14699	20699
max clock frequency (MHz)	105.83	75.55
encryption speed (Gbps)	4.23	3.02

we used in the real-time encryption and decryption experiment has a 27 MHz clock frequency.

## 4 Summary

We have reviewed the cellular automaton-based PRNGs, and have shown that one of these PRNGs, which was denoted as CA5-535945230, can generate high-quality random numbers which can pass all of the NIST statistical tests. We demonstrated that the encryption algorithm using the CA-based PRNG has a 3 Gbps encryption speed in the case of FPGA (20 Kgate used). This suggests that CA is suitable for developing a high-speed hardwarelike encryption system.

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